

**GDR 134**

**GROUPEMENT DE RECHERCHE  
TRAITEMENT DU SIGNAL ET IMAGES**



WAVEL TO K  
REFERENCE II

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<b>angiography :</b>	GDR "angiographie".
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# **Digital Signal Processing Tools**

## Contents

### Graphics

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### Design

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<b>phasevar :</b>	Phase variation
<b>zremez :</b>	Optimal Chebyshev FIR filter

## **allres**

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### **Purpose**

Plot all responses

### **Synopsis**

allres(H)

### **Description**

allres creates a figure that allows the user to plot impulse, magnitude, phase, group delay responses and zero plot of any FIR filter interactively.

### **See also**

IMPRES, MAGRES, PHASERES, GDRES, ZPLOT in the GDR Wavelet Toolbox.

### **Author**

Olivier Rioul, March 1994

## gdres

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### Purpose

Group delay response

### Synopsis

gdres(H1)

gdres(H1,H2,H3,H4,H5,H6,H7,H8,H9)

### Description

gdres(H) plots the group delay response of FIR filter H. If H is a matrix, then multiple responses are plotted for each line.

gdres(H1,H2,...) is another way of producing multiple responses on the same plot, except that filters may have different lengths.

### Algorithm

The group delay response is computed using the following formula:

$$G_d(w) = -\frac{d(\arg(H(w)))}{dw}$$

### See also

ALLRES, PHASERES in the GDR Wavelet Toolbox.

### Author

Olivier Rioul, March 1994

## **impres**

---

### **Purpose**

Impulse response

### **Synopsis**

`impres(H)`

`impres(H1,H2,...)`

### **Description**

`impres(H)` plots the FIR filter coefficients  $H$  as 'stems' from the x-axis. If  $H$  is a matrix, then multiple responses are plotted for each line.

`impres(H1,H2,...)` is another way of producing multiple responses on the same figure, except that filters may have different lengths.

### **See also**

STEM in the standard MATLAB Toolbox and ALLRES in the GDR Wavelet Toolbox.

### **Author**

Olivier Rioul, March 1994

## magres

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### Purpose

Magnitude frequency response

### Synopsis

magres(H)

magres(H1,H2,...)

magres(...,'dB')

magres(...,'dB2')

magres(square(H),'dB2')

magres(...,dBmin)

### Description

magres(H) plots the magnitude frequency response of FIR filter H using linear y-axis scale. If H is a matrix, then multiple responses are plotted for each line.

magres(H1,H2,...) is another way of producing multiple responses on the same plot, except that filters may have different lengths.

magres(...,'dB') plots magnitude response(s) in dB, assuming pass-band gain = 1.

magres(...,'dB2') assumes filter(s) are convolutional squares:

magres(square(H),'dB2') is equivalent to MAGRES(H,'dB')

magres(...,dBmin) selects plot in dB and sets dBmin < 0 as the minimum attenuation shown on the figure. Default is dBmin = -40dB.

### See also

ALLRES, SQUARE in the GDR Wavelet Toolbox.

### Author

Olivier Rioul, March 1994

## **mirror**

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### **Purpose**

Mirror filter design

### **Synopsis**

mirror(H)

G=mirror(H)

### **Description**

mirror(H) is the 'mirror' filter of FIR filter H. It applies linewise if H is a matrix

### **Algorithm**

Given a low-pass (high-pass) FIR filter h whose length is L. Its high-pass (low-pass) FIR mirror filter is given by

$$g(n)=(-1)^nh(L-1-n), n=0,1,\dots,L-1.$$

### **Author**

Olivier Rioul, March 1994

## phaseres

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### **Purpose**

Phase frequency response

### **Synopsis**

phaseres(H)

phaseres(H1,H2,...,H9)

### **Description**

phaseres(H) plots the phase frequency response of FIR filter H. If H is a matrix, then multiple responses are plotted for each line.

phaseres(H1,H2,...) is another way of producing multiple responses on the same plot, except that filters may have different lengths.

### **See also**

ALLRES, GDRES in the GDR Wavelet Toolbox.

### **Author**

Olivier Rioul, March 1994

## **phasevar**

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### **Purpose**

Phase variation of half-band FIR filter

### **Synopsis**

phasevar(H)

### **Description**

phasevar(H) gives total group delay variation of FIR half-band low-pass or high-pass filter H in the pass-band. It is a (positive) measure of phase distortion between 0 and L-1 (samples). If it is 0, then H is linear phase. Check the result with GDRES. If H is a matrix, phasevar(H) is a column vector containing the group delay variation for each line.

### **See also**

PHASERES, FACTOR, GDRES in the GDR Wavelet Toolbox.

### **Author**

Olivier Rioul, March 1994

## square

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### Purpose

Convolutional square

### Synopsis

$p = \text{square}(H)$

### Description

$\text{square}(H)$  is the convolutional square of FIR filter  $H$ , whose phase response is linear. Applies line-wise if  $H$  is a matrix

### Author

Olivier Rioul, March 1994

## **zplot**

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### **Purpose**

Plots zeroes of FIR filter H in the complex plane

### **Synopsis**

`zplot(H)`

`zplot(H1,H2,...,H9)`

### **Description**

`zplot(H)` plots zeroes of FIR filter H in the complex plane. If H is a matrix, then zeroes are plotted for each line.

`zplot(H1,H2,...)` is another way of producing multiple zero plots on the same figure, except that filters may have different lengths.

### **See also**

ALLRES in the GDR Wavelet Toolbox.

### **Author**

Olivier Rioul, March 1994

## **zremez**

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### **Purpose**

Optimal Chebyshev filter design

### **Synopsis**

$[H,w]=zremez(L,K,B,dw,p,w)$

### **Description**

Given number of taps  $L$ , flatness order  $K$  (number of zeros at  $z=-1$ ), normalized transition bandwidth  $B$ , transition bandwidth offset  $dw$  about 0.25 (positive or negative), and pass-band weight  $p=\delta_2/\delta_1$ ,  $ZREMEZ(L,K,B,dw,p)$  gives a vector containing the coefficients of linear-phase FIR low-pass filter that is optimal in the Chebyshev sense for these specifications. Its mirror filter (see **MIRROR**) is the corresponding high-pass filter for the same specifications except that  $dw$  is changed to  $-dw$ .

$[H,w]=zremez(...)$  gives access to alternance values  $w_0, \dots, w_N$  in the grid  $0:64*N$ .

$zremez(L,K,B,dw,p,w)$  uses alternance values  $w$  as first guess.

**Note** : This program includes **REMEZ** in the Signal Processing Toolbox as a special case.

### **See also**

**MIRROR**, **REMEZWAV** in the GDR Wavelet Toolbox.

### **Author**

Olivier Rioul, March 1994

## **Wavelet Tools**

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### Orthogonal wavelet

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**wfilter** : Frequency selective regular wavelet filter

### Bi-orthogonal wavelets

**co** : Bi-orthogonal complement

### Graphical tools

**wlet** : Wavelet, scaling function and derivatives

### Regularity estimates

**reg** : Hölder regularity order  
**sobreg** : Sobolev regularity order

## Purpose

Bi-orthogonal Complement filter

## Synopsis

$H_p = \text{co}(H, K)$

## Description

$\text{co}(H)$  is the bi-orthogonal complement filter of low-pass linear-phase  $L$ -tap filter  $H$ . Its length is  $L$  if  $L$  is even (type A) and  $L-2$  if  $L$  is odd (type B). Its pass-band gain is  $2/g$  where  $g$  is the pass-band gain of  $H$ .

$\text{co}(H, K)$  forces  $K$  zeros at  $z = -1$  in the complement filter. Its length will be  $L + 2*(K-1)$  where  $L$  is the length of  $H$ .  $\text{CO}(H)$  is equivalent to  $\text{CO}(H, 0)$  if  $L$  is odd,  $\text{CO}(H, 1)$  if  $L$  is even. In all cases,  $L-K$  must be odd.

## Author

Olivier Rioul, March 1994

## References

[1] Olivier Rioul, Ondelettes regulieres: Application a la compression d'images fixes, PhD Thesis, Telecom Paris, March 1993.

## daub

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### Purpose

Daubechies filter

### Synopsis

H=daub(L,type)

### Description

daub(L) or daub(L,'min') produces the minimum phase Daubechies low-pass filter of length L.

daub(L,'lin') produces the Daubechies filter whose phase response is closest to linear (as determined by PHASEVAR).

### Algorithm

In the frequency domain, the wavelet,  $\psi$ , is defined as

$$\psi(\omega) = H_1(e^{j\frac{\omega}{2}})\phi(\omega)$$

where

$$\phi(\omega) = \prod_{k=2} H_0(e^{j\frac{\omega}{2^k}})$$

is the scaling function.  $H_0$  is the low-pass filter and  $H_1$  is its high-pass mirror filter. Daubechies low-pass filter  $H_0$  is designed such that it has the maximum number of zeros at  $z=-1$ .

### See also

FACTOR, PHASEVAR, REMEZWAV, WFILTER in the GDR Wavelet Toolbox

### Author

Olivier Rioul, March 1994

### References

[1] Ingrid Daubechies, Orthonormal bases of compactly supported wavelets, Comm. Pure Appl. Math., Vol. XLI, No. 7, pp. 909-996, 1988.

[2] Ingrid Daubechies, Orthonormal bases of compactly supported wavelets II, Variations on a theme, SIAM J. Math. Anal., Vol. 24, 1993.

## factor

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### Purpose

Zero factoring (Riesz lemma)

### Synopsis

$H = \text{factor}(Z, \text{type})$

### Description

$\text{factor}(Z)$  produces filter  $H$  by factoring the complex zeroes  $Z$  of the linear phase filter  $\text{SQUARE}(H)$  (as obtained by design routines). Zeroes in  $Z$  must have even multiplicity on the unit circle and be associated in  $(z, z^*, 1/z, 1/z^*)$  quadruples outside the unit circle. The resulting filter  $H$  is normalized such that  $\text{NORM}(H) = 1$ . By default, selection of zeros  $(z, z^*)$  or  $(1/z, 1/z^*)$  for  $H$  is done manually by clicking the mouse on the screen.

$\text{factor}(Z, 'min')$  gives the minimum phase solution  $H$

$\text{factor}(Z, 'max')$  gives the maximum phase solution  $H$

$\text{factor}(Z, 'lin')$  gives the solution whose phase response is closest to linear (as determined by  $\text{PHASEVAR}$ ).

$\text{factor}(Z, 'all')$  gives all solutions as lines of matrix  $H = [H_0; H_1; H_2; \dots]$  With the convention that indexes range from 0,  $H_n$  is designed by selecting zeros outside (1) or inside (0) the unit disk as absolute angle of zeros increases. For example,  $n = 1011$  in base 2 means (in reverse order): First outside, then outside, then inside, then outside.  $H_0$  is the minimum-phase solution, while last  $H_n$  ( $n = 2^{(\cdot)} - 1$ ) is the maximum-phase solution. Note that the second half of solutions are the time-reversals of the first half.

$\text{factor}(Z, 'allsorted')$  gives all solutions sorted from increasing  $\text{PHASEVAR}$ s as lines of matrix  $H$ .

### See also

$\text{PHASEVAR}$ ,  $\text{REMEZWAV}$ ,  $\text{MAGRES}$ ,  $\text{SQUARE}$ ,  $\text{ZPLOT}$  in the GDR Wavelet Toolbox and  $\text{NORM}$  in the standard MATLAB Toolbox.

### Author

Olivier Rioul, March 1994

### References

- [1] Mark J. T. Smith and Thomas P. Barnwell III, Exact reconstruction techniques for tree-structured subband coders, *IEEE Trans. ASSP*, Vol. 34, No. 3, pp. 434-441, 1986.
- [2] Ingrid Daubechies, Orthonormal bases of compactly supported wavelets, *Comm. Pure Appl. Math.*, Vol. XLI, No. 7, pp. 909-996, 1988.
- [3] Olivier Rioul and Pierre Duhamel, A Remez exchange algorithm for orthonormal wavelets, *IEEE Trans. CAS II*, To appear.

## reg

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### Purpose

Hölder Regularity order of low-pass filter H.

### Synopsis

$r = \text{reg}(H, K)$

### Description

$\text{reg}(H)$  gives a sharp upper bound for the Hölder regularity order of low-pass filter H (it may be negative).

$\text{reg}(H, K)$  forces the number of zeros at  $z = -1$  in H to K. By default, K is estimated by successive division with remainders  $< 1e-3$

### Algorithm

In order to ensure that the infinite product (scaling function)

$$\phi(\omega) = \prod_{k=1}^{\infty} H_0\left(\frac{e^{j\omega}}{2^k}\right)$$

converges to a smooth function rather than breaks into fractals,  $H_0(\omega)$  should have some regularity, that is a certain number of continuous derivatives. The regularity is shown to increase with the number of zeros at  $z = -1$ . In reference [1], two technics are developed to compute this regularity: the Hölder regularity and the sobolev regularity.

### See also

SOBREG in the GDR Wavelet Toolbox

### Author

Olivier Rioul, March 1994

### References

[1] Olivier Rioul, Simple regularity criteria for subdivision schemes, SIAM J. Math. Anal., Vol. 23, No. 6, pp. 1544-1576, Nov. 1992.

## sobreg

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### Purpose

Optimal Sobolev regularity order of low-pass filter H

### Syopsis

$r = \text{sobreg}(H, K)$

### Description

$\text{sobreg}(H)$  gives the optimal Sobolev regularity order of low-pass filter H (it may be negative).

$\text{sobreg}(H, K)$  forces the number of zeros at  $z = -1$  in H to K. By default, K is estimated by successive division with remainders  $< 1e-3$ .

### Algorithm

See REG in the GDR Wavelet toolbox

### See also

REG in the GDR Wavelet Toolbox

### Author

Olivier Rioul, March 1994

### References

[1] Olivier Rioul, Simple regularity criteria for subdivision schemes, SIAM J. Math. Anal., Vol. 23, No. 6, pp. 1544-1576, Nov. 1992.

## remezwav

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### Purpose

Optimal wavelet generation using Remez exchange algorithm

### Synopsis

$[p,r,delta]=remezwav(L,K,B)$

### Description

$remezwav(L,K,B)$  gives impulse response of maximally frequency selective  $P(z)$ , product filter of paraunitary filter bank solution  $H(z)$  of length  $L$  satisfying  $K$  flatness constraints (wavelet filter). The normalized transition bandwidth is given as  $B$ . This argument is optional if  $K=L/2$ .

$[P,R]=REMEZWAV(L,K,B)$  also gives the roots of  $P(z)$ , which can be used to determine  $H(z)$ .

$[P,R,DELTA]=REMEZWAV(L,K,B)$  also gives maximum deviation, if  $K < L/2$  (i.e.,  $P(z)$  is between 0 and delta in stop band).

### Algorithm

The algorithm of `REMEZWAV` is a generalization of the algorithm `REMEZ` we can find in Matlab Toolbox. It is developed in details in reference [1].

### Author

Olivier Rioul, November 1992.

### References

- [1] Olivier Rioul and Pierre Duhamel, A Remez exchange algorithm for orthonormal wavelets, IEEE Trans. CAS II, To appear.
- [2] Ingrid Daubechies, Orthonormal bases of compactly supported wavelets, Comm. Pure Appl. Math., Vol. XLI, No. 7, pp. 909-996, 1988.
- [3] Mark J. T. Smith and Thomas P. Barnwell III, Exact reconstruction techniques for tree-structured subband coders, IEEE Trans. ASSP, Vol. 34, No. 3, pp. 434-441, 1986.

## wfilter

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### Purpose

Orthonormal wavelet filter

### Synopsis

wfilter(L,K,B)

wfilter(L,K,B,'min')

### Description

wfilter(L,K,B) or wfilter(L,K,B,'min') produces the minimum-phase orthonormal wavelet low-pass filter of length L with K zeros at the Nyquist frequency ( $z=-1$ ), and maximally frequency selective given normalized transition bandwidth B. (See REMEZWAV)

wfilter(L,K,B,'lin') produces the wavelet filter whose phase response is closest to linear (as determined by PHASEVAR)

wfilter(L,L/2,B,'type') is equivalent to DAUB(L,'type')

### Algorithm

See DAUB in GDR Wavelet Toolbox

### See also

DAUB, FACTOR, PHASEVAR, REMEZWAV from the GDR Wavelet Toolbox

### Author

Olivier Rioul, March 1994

### References

- [1] Olivier Rioul and Pierre Duhamel, A Remez exchange algorithm for orthonormal wavelets, IEEE Trans. CAS II, To appear.
- [2] Ingrid Daubechies, Orthonormal bases of compactly supported wavelets, Comm. Pure Appl. Math., Vol. XLI, No. 7, pp. 909-996, 1988.
- [3] Mark J. T. Smith and Thomas P. Barnwell III, Exact reconstruction techniques for tree-structured subband coders, IEEE Trans. ASSP, Vol. 34, No. 3, pp. 434-441, 1986.

## wlet

---

### **Purpose**

Plots wavelets, scaling functions and their derivatives

### **Synopsis**

wlet(H,iter,dn)

### **Description**

wlet(H) plots scaling function or wavelet associated to filter H, if H is a low-pass or high-pass filter, respectively.

wlet(H,n) forces the number of iterations to n. By default, n is determined such that the curve has at least 200 points.

wlet(H,n,m) plots the m-th derivative of scaling function or wavelet (see above) using n iterations.

### **Author**

Olivier Rioul, March 1994

## **Image Processing Tools**

## Contents

### I/O functions

**rawread** : Read bytes from binary file.  
**rawwrite** : Write bytes to binary file.

### Visualization

**visu** : Visualizes intensity pixels.  
**visuerr** : Visualizes error image.

### Coding Tools

**polish** : Polish image data for output.  
**histogram** : Plot histogram or compute pdf.  
**entropy** : Zero-th order entropy.  
**huffmann** : Global huffmann coding rate.  
**snr** : Signal to noise ratio.

## entropy

---

### Purpose

Compute the 0th-order entropy of image I (in bits per pixel)

### Synopsis

`s=entropy(p)`

### Description

`entropy(p)` compute the 0th-order entropy of image I (in bits per pixel) (pre-calculation of its pdf is needed).

`entropy(P)` does the same given the pdf P of I (see HISTOGRAM).

### See also

HISTOGRAM, HUFFMANN in the GDR Wavelet Toolbox.

### Author

Olivier Rioul, March 1994

## **histo**

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### **Purpose**

Performs the inner calculations used by HISTOGRAM.

### **Synopsis**

n=histo(x,nn)

### **Description**

performs the inner calculations used by HISTOGRAM.

### **Author**

Olivier Rioul, March 1994

## histogram

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### Purpose

Plot histogram of image or compute its pdf

### Synopsis

`p=histogram(I,flag)`

### Description

`histogram` plots histogram of image or compute its pdf.

`histogram(I)` returns the pdf  $P$  of image  $I$  (or any set of values), where  $P(i)$  is the percentage of pixels having value  $\min(I)+i-1$ .

`histogram(I,'plot')` also plots the histogram of  $I$ .

**Caution** : The pixel values are first rounded to integers.

### Author

Olivier Rioul, March 1994

## **huffmann**

---

### **Purpose**

Huffman coding

### **Synopsis**

$h = \text{huffmann}(p)$

### **Description**

$\text{huffmann}(I)$  gives the number of bits per pixel obtained using a global Huffman code (GHC) on image  $I$  (precalculation of its pdf is needed).

$\text{huffmann}(P)$  does the same given the pdf  $P$  of  $I$  (See HISTOGRAM). It takes a few seconds when  $\text{length}(P)=256$ .

### **See also**

ENTROPY, HISTOGRAM in the GDR Wavelet Toolbox

### **Author**

Olivier Rioul, March 1994

## **polish**

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### **Purpose**

Round to integers between 0 and 255

### **Synopsis**

`x=polish(x)`

### **Description**

`polish(X)` returns matrix or vector `X` with values rounded to integers between 0 and 255 (Values above 255 are rounded to 255 and negative values are rounded to 0).

### **See also**

VISU in the GDR Wavelet Toolbox

### **Author**

Gerard Yvon, March 1994

## **rawread**

---

### **Purpose**

Read a specified file

### **Synopsis**

`I = rawread(filename,N,Nskip)`

### **Description**

`rawread('filename',[nl,nc])` reads specified file containing a matrix of `nl*nc` bytes in "raw" format (bytes stored line by line).

`rawread('filename',nc)` does not use information on the number of lines but reads until the end-of-file is reached.

`rawread('filename',[nl,nc],nskip)` or `RAWREAD('filename',nc,skip)` specify that a header of `nskip` bytes has to be skipped at the beginning of the file.

### **See also**

`RAWWRITE` in the GDR Wavelet Toolbox.

### **Author**

Gerard Yvon, March 1994

## **rawwrite**

---

### **Purpose**

Write bytes from a matrix to a filename

### **Synopsis**

```
rawwrite(I,filename)
```

### **Description**

`rawwrite(I,'filename')` writes bytes from matrix `I` to the specified filename in "raw" format (bytes stored line by line).

**Caution:** Values from matrix `I` must be POLISH'ed first.

### **See also**

POLISH, RAWREAD in the GDR Wavelet Toolbox.

### **Author**

Olivier Rioul, March 1994

## **snr**

---

### **Purpose**

Signal to Noise Ratio

### **Synopsis**

$r = \text{snr}(I, I_p)$

### **Description**

$\text{snr}(I, I_p)$  gives the signal to noise ratio in dB for original image  $I$  and processed image  $I_p$ .

$\text{snr}(I - I_p)$  gives the Peak Signal over Noise Ratio (PSNR) when  $I$  is originally on 8 bits per pixel.

### **Author**

Olivier Rioul, March 1994

## **visu**

---

### **Purpose**

Image visualization (gray levels)

### **Synopsis**

`visu(I,zm)`

### **Description**

`visu(I)` displays intensity pixels of image `I` on the bottom-left of the screen using gray colormap. By default, the image is displayed at the resolution of the screen. The image pixels are assumed to be encoded on 8 bits (from 0 to 255). It is a good idea to POLISH them first.

`visu(I,zoom)` specifies a zoom factor (magnifies if `zoom>1`). If `zoom` is negative, the image is displayed in reverse video. Popup menu 'zoom' specifies the resolution at which the image can be displayed: 'zoom 1' always sets back to the screen resolution. Popup menu 'local zoom n' allows to enable or inhibit the Matlab built-in function 'zoom'.

### **See also**

POLISH, VISUERR from the GDR Wavelet Toolbox and ZOOM in the standard MATLAB Toolbox.

### **Author**

Olivier Rioul, March 1994

## **visuerr**

---

### **Purpose**

Displays absolute values of intensity pixels of an "error image"

### **Synopsis**

`visuerr(x, zoom)`

### **Description**

`visuerr(I)` displays absolute values of intensity pixels of an "error image" I. The absolute values of pixels are fitted in the interval [black->white] (black correspond to zero).

`visuerr(I, zoom)` displays the "error image" with the specified zoom factor. If zoom is negative, the image is displayed in reverse video.

### **See also**

VISU in the GDR Wavelet Toolbox

### **Author**

Olivier Rioul, March 1994

## **2D Wavelet Transformation**

## Contents

### Periodic wavelet transform

**psplit2** : One step of wavelet analysis.  
**pmerge2** : One step of wavelet synthesis.  
**pdwt2** : Wavelet transform.  
**pidwt2** : Inverse wavelet transform.

### Miscellaneous

**sub** : Select subimage.  
**wvisu** : Visualize wavelet coefficients.

### Quantization

**wq** : Quantization of wavelet coefficients.  
**iwq** : Inverse quantization of wavelet coefficients.

### Coding

**wbitrate** : Total bitrate of wavelet coefficients.

### Optimal bit allocation

**rdoptim** : Rate/Distortion optimisation of quantizers.  
**rdoptim1** : Initialisation for rdoptim.

## **iwq**

---

### **Purpose**

Inverse quantization of wavelet coefficients

### **Synopsis**

$W_q = \text{iwq}(\text{fname}, W_q, c)$

### **Description**

$\text{iwq}(\text{'fname'}, W_q, c)$  rescales subimages  $(j,k)$  of wavelet matrix  $W_q$  after quantization, using scaling array  $c(j,k)$  (see SQ,ISQ) and quantizer 'fname' (see ISQ).

### **See also**

ISQ, SQ in the GDR Wavelet Toolbox.

### **Author**

Olivier Rioul, March 1994

## pdwt2

---

### Purpose

Orthonormal 2D separable wavelet transform with periodic extension

### Synopsis

$[I,J]=pdwt2(I,g,J)$

### Description

$pdwt2(I,H,j)$  performs an orthonormal discrete wavelet transform (DWT) of image  $I$  on  $j$  levels ("octaves") using separable filters based on low-pass orthonormal (synthesis) filter  $H$  and periodic extension at the edges. The resulting matrix is the "canonical form" (see  $PSPLIT2$ ).

$pdwt2(I,H)$  does the wavelet decomposition with maximum number of levels  $j$  such that the image dimensions stay even and greater than the filter length. This  $j$  is returned as a second argument.

### See also

$PSPLIT2$ ,  $PIDWT2$  in the GDR Wavelet Toolbox.

### Author

Olivier Rioul, March 1994

## pidwt2

---

### Purpose

Orthonormal 2D inverse wavelet transform with periodic extension

### Synopsis

$I = \text{pidwt2}(I, g, J)$

### Description

PIDWT2(W,H,j) performs an orthonormal inverse discrete wavelet transform (IDWT) of image W on j levels ("octaves") using separable filters based on low-pass orthonormal (synthesis) filter H and periodic extension at the edges. The input matrix W must be in the "canonical form" (see PMERGE2).

### See also

PSPLIT2, PDWT2 in the GDR Wavelet Toolbox.

### Author

Olivier Rioul, March 1994

## **pmerge2**

---

### **Purpose**

Periodic separable wavelet merge

### **Synopsis**

$I = \text{pmerge2}(I, g)$

### **Description**

$\text{pmerge2}(I, H)$  performs one step of orthonormal wavelet reconstruction on the lines and columns of  $I$ , using low-pass orthonormal (synthesis) filter  $H$  and periodic extension at the edges. The input matrix must be in the form  $[\text{low}, \text{vert}; \text{horiz}, \text{diag}]$  (see  $\text{PSPLIT2}$ ).

### **See also**

$\text{PMERGE2}$ ,  $\text{PIDWT2}$  in the GDR Wavelet Toolbox.

### **Author**

Olivier Rioul, March 1994

## **pmergec2**

---

### **Purpose**

Periodic wavelet merge column by column

### **Synopsis**

$I = \text{pmergec2}(I, g)$

### **Description**

PMERGE2(I,H) performs one step of orthonormal wavelet reconstruction on the columns of I using low-pass orthonormal (synthesis) filter H and periodic extension at the edges. The input matrix must be in the form [lowpass; highpass].

### **See also**

PMERGE2 in the GDR Wavelet Toolbox

### **Author**

Olivier Rioul, March 1994

## psplit2

---

### Purpose

Periodic separable wavelet split

### Synopsis

I=psplit2(I,g)

### Description

PSPLIT2(I,H) performs one step of orthonormal wavelet decomposition on the columns and lines of I, using low-pass orthonormal (synthesis) filter H and periodic extension at the edges. The resulting matrix is in the form [low , vert ; horiz , diag] where:  
subimage "low" = horizontal and vertical low-pass component  
subimage "vert" = horizontal high-pass and vertical low-pass component  
subimage "horiz" = horizontal low-pass and vertical high-pass component  
subimage "diag" = horizontal and vertical high-pass component

### See also

PSPLITC2, PDWT2 in the GDR Wavelet Toolbox

### Author

Olivier Rioul, March 1994

## **psplitc2**

---

### **Purpose**

Periodic wavelet split column by column

### **Synopsis**

`I=psplitc2(I,g)`

### **Description**

`psplitc2(I,H)` performs one step of orthonormal wavelet decomposition on the columns of `I` using low-pass orthonormal (synthesis) filter `H` and periodic extension at the edges. The result matrix is in the form `[lowpass; highpass]`.

### **See also**

PSPLIT2 in the GDR Wavelet Toolbox

### **Author**

Olivier Rioul, March 1994

## **rdoptim**

---

### **Purpose**

Rate/distortion optimisation

### **Synopsis**

`[bpp,psnr,ar]=rdoptim(dbpp1,dbpp2)`

### **Description**

After initialization by `RDOPTIM1`, which specifies the image and wavelet filter, `[bpp,psnr,bpparray]=rdoptim(dbpp)` performs rate/distortion optimization for desired bit rate = `dbpp` (in bits/pixel) and gives `bpp`, the actual bit rate obtained, `psnr`, an estimated value of the Peak SNR at reconstruction, and `bpparray`, the optimal array of bits per pixel for each subimage with the optimal number of decompositions.

`rdoptim(dbpp1,dbpp2)` plots the optimal psnr/bit rate curve for bit rates between `dbpp1` and `dbpp2`.

`[BPP,PSNR,JOPT]=rdoptim(dbpp1,dbpp2)` also gives the corresponding values of `psnr`, bit rates and optimal numbers of decompositions.

### **See also**

`RDOPTIM1` in the GDR Wavelet Toolbox.

### **Author**

Olivier Rioul, March 1994

## **rdoptim1**

---

### **Purpose**

Initialisation for RDOPTIM

### **Synopsis**

```
rdoptim1(I,h,fname,minbpp,maxbpp,Nch)
```

### **Description**

RDOPTIM(I,H,'fname',minbpp,maxbpp,Nch) does initialisation for image I, orthonormal wavelet filter H, Nch different quantizers for each subimage, with bits/pixel ranging from minbpp to maxbpp. minbpp and maxbpp are arrays indexed by (j=1:Jmax,k=0:3) of min and max bpps for subimage(j,k). The bit rate criterion is given by 'fname': If it is 'bpp', the bit rate is that allocated in the quantizers. Otherwise it should be the name of a MATLAB function giving bitrates (see ENTROPY, HUFFMANN).

### **See also**

ENTROPY, HUFFMANN, RDOPTIM in the GDR Wavelet Toolbox.

### **Author**

Olivier Rioul, March 1994

### **References**

[1] Olivier Rioul, Ondelettes regulieres: Application a la compression d'images fixes, PhD Thesis, Telecom Paris, March 1993.

## **rdoptim2**

---

### **Purpose**

Inner calculations used by RDOPTIM

### **Synopsis**

```
[r,d,JJopt,indd]=rdoptim2(R,D,s)
```

### **Description**

rdoptim2 performs the inner calculations used by RDOPTIM.

### **See also**

RDOPTIM in the GDR Wavelet Toolbox.

### **Author**

Olivier Rioul, March 1994

## sub

---

### Purpose

Select sub-image

### Synopsis

$A = \text{sub}(S, j, k)$

### Description

$\text{sub}(W, j, k)$  or  $\text{sub}(W, jk)$  returns sub-image of wavelet matrix  $W$  (as returned by PDWT2) at level  $j$  and orientation  $k$ , where:

$k=0$ : low-pass component if  $j$  is the maximum number of levels

$k=1$ : vertical component,  $k=2$ : horizontal component,  $k=3$ : diagonal component

$\text{sub}(\text{size}(W), j, k)$  or  $\text{sub}(\text{size}(W), jk)$  returns index matrix  $A$  such that  $W(A)$  is the sub-image of wavelet matrix  $W$  at level  $j$  and orientation  $k$ .

$w(\text{sub}(\text{size}(W), jk))$  is equivalent to  $\text{sub}(W, jk)$  but the former can be used for indexing  $w$  at the lhs of an expression.

### See also

PDWT2 in the GDR Wavelet Toolbox

### Author

Olivier Rioul, March 1994

## wbtrate

---

### Purpose

Total bitrate of wavelet matrix

### Synopsis

```
[s, t]=wbtrate(fname,W,J)
```

### Description

wbtrate('fname',W,j) computes the total bitrate necessary to encode quantized wavelet matrix W on j levels. The bitrate (in bits per pixel) is computed for each subimage w using fname(w) (see ENTROPY and HUFFMANN). It returns the array of bitrates as a second argument: a matrix indexed by (j=1:J,k=0:3) which gives the bitrate for subimage (j,k) (see SUB)

### See also

ENTROPY, HUFFMANN, SUB in the GDR Wavelet Toolbox

### Author

Olivier Rioul, March 1994

## wq

---

### **Purpose**

Quantization of wavelet coefficients

### **Synopsis**

$[W,c]=wq(fname,W,n)$

### **Description**

$[Wq,c]=wq('fname',W,n)$  quantizes subimages (j,k) of wavelet matrix W on n(j,k) bits per pixel using quantizer 'fname' (see SQ) and returns scaling array c(j,k) as second argument (see SQ).

### **See also**

SQ in the GDR Wavelet Toolbox.

### **Author**

Olivier Rioul, March 1994

## **wvisu**

---

### **Purpose**

Displays wavelet matrix on the screen

### **Synopsis**

wvisu(W,J,zoom)

### **Description**

WVISU(W,J) or WVISU(W,J,zoom) does VISUERR by re-scaling each subimage of wavelet matrix W for better visibility. If the zoom factor is negative, wavelet coefficients are displayed in reverse video (but not the low-pass component).

### **See also**

VISU, VISUERR in the GDR Wavelet Toolbox

### **Author**

Olivier Rioul, March 1994

## **Image Quantization**

## Contents

<b>sq :</b>	Uniform scalar quantization
<b>isq :</b>	Inverse uniform scalar quantization

## isq

---

### Purpose

Inverse uniform scalar quantization

### Synopsis

$I = \text{isq}(I, c)$

### Description

$\text{isq}(I, c)$  rescales the pixels values in  $I$  after quantization, using:  
 $c = a + i * b$  as given by SQ.

After inverse scalar quantization, any zero value will be mapped to zero.

### See also

SQ in the GDR Wavelet Toolbox

### Author

Olivier Rioul, March 1994

## sq

---

### Purpose

uniform scalar quantization

### Synopsis

$[I,c]=sq(I,n)$

### Description

$[I_q,c]=SQ(I,n)$  quantifies  $I$  on  $n$  bits/pixel. The output values in  $I_q$  lie between 0 and  $\text{round}(2^n)-1$ . Scalar  $c=a+i*b$  gives constants  $a$  and  $b$  such that  $I_q=\text{round}(a*I+b)$ . After inverse scalar quantization, any zero value will be mapped to zero.

### See also

ISQ in the GDR Wavelet Toolbox

### Author

Olivier Rioul, March 1994